ALTERNATING CURRENT

$$\boxed{V = V_0 \sin(\underbrace{\overbrace{\omega}^{\text{Angular freq.}} t + \overbrace{\varphi}^{\text{Initial phase}}}_{\text{Phase}})}$$

V: (Instantaneous Voltage)

V_0: Voltage Amplitude (Peak voltage)

$$\left\{ P_{rms} = \frac{1}{(t_2 - t_1)} \int_{t_1}^{t_2} P(t)\, dt \right\}$$

(Any property P)

$$V_{rms}^2 = \frac{\int_t^{T+t} V^2 \sin^2(\omega t + \varphi)\, dt}{(T+t) - t}$$

$$= \frac{V^2}{2T} \int_t^{T+t} 1 - \cos(2\omega t + 2\varphi)\, dt$$

$$= \frac{V^2}{2T} \left[t - \frac{\sin(2\omega t + 2\varphi)}{2\omega} \right]_t^{T+t} = \frac{V^2}{2}$$

$$\Rightarrow \boxed{V_{rms} = \frac{V}{\sqrt{2}}}$$

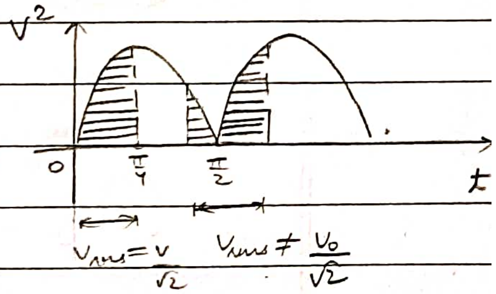
V_{rms} is also called equivalent D.C Voltage

REMARK: Unless mentioned, volt. & current given for AC circuit are rms values.

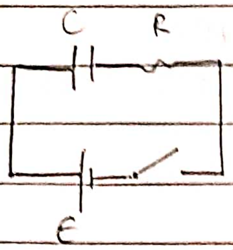
NOTE: In Time period → V_{rms}

Time period	V_{rms}
T	$V_0/\sqrt{2}$
$T/2$	$V_0/\sqrt{2}$
$T/4$	$V_0/\sqrt{2}$ only for 1 case

since



Q.



Switch closed at $t=0$.
Find i_{rms} for $t = 0$ to RC

A. $E - \frac{q}{C} - iR = 0 \Rightarrow R\frac{dq}{dt} + \frac{q}{C} = E$

$$\Rightarrow q = EC\left(1 - e^{-\frac{t}{RC}}\right)$$

$$\Rightarrow i = \frac{E}{R} e^{-t/RC}$$

$$i_{rms}^2 = \left(\frac{\int_0^{RC} \frac{E}{R} e^{-t/RC} \, dt}{RC}\right)$$

$V = V_0 \sin \omega t$

$i = i_0 \sin(\omega t + \varphi)$ $\Big\} \Rightarrow P = Vi$

$$P_{avg} = \frac{\int_0^T V_0 \sin \omega t \cdot i_0 \sin(\omega t + \varphi) \, dt}{T}$$

$$= \frac{i_0 V_0}{2T} \int_0^T \cos\varphi - \cos(2\omega t + \varphi) \, dt$$

$$\Rightarrow \boxed{P_{avg} = \frac{i_0 V_0 \cos\varphi}{2}} = \left(\frac{i_0}{\sqrt{2}}\right)\left(\frac{V_0}{\sqrt{2}}\right)\cos\varphi$$

$$= \boxed{V_{rms}\, i_{rms} \cos\varphi}$$

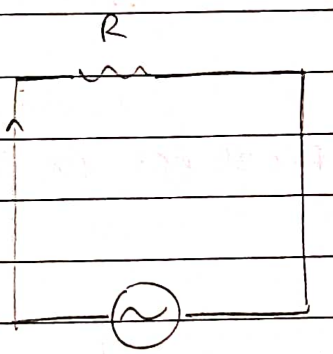
(Power factor of AC circuit)

if $\cos\varphi = 0 \Rightarrow P_{avg} = 0 \Rightarrow$ wattless current

Wattless component of current – $i \sin(\varphi)$

($\because$ only $i\cos(\varphi)$ dissipates power)

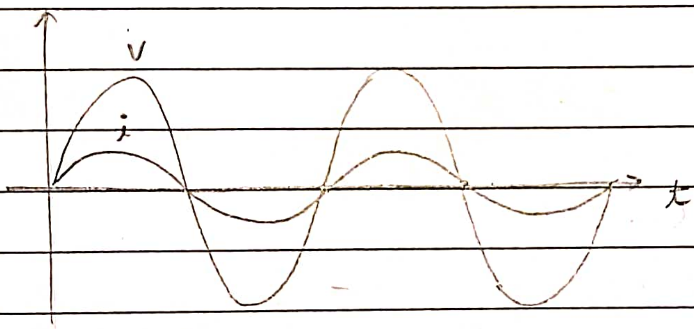
→ <u>Resistor</u>



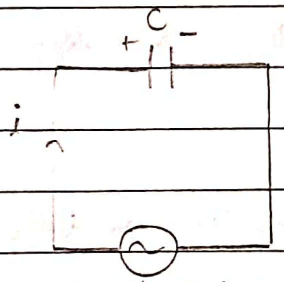
$$V - iR = 0$$

$$\Rightarrow \quad i = \left(\frac{V_0}{R}\right) \sin\omega t$$

$\Rightarrow$ V & i in same phase!



→ <u>Capacitor</u>

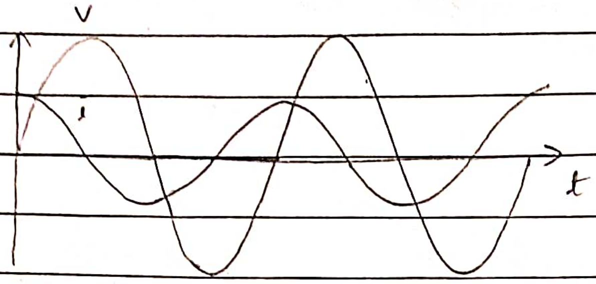


$$V - \frac{q}{C} = 0$$

$$\Rightarrow \quad q = V_0 C \sin\omega t$$

$$\Rightarrow \quad i = \frac{dq}{dt} = V_0 C\omega \cos\omega t = \frac{V_0}{\left(\frac{1}{\omega C}\right)} \sin\left(\omega t + \frac{\pi}{2}\right)$$

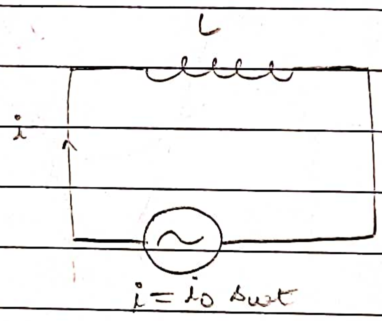
$\Rightarrow$ i leads V by $\pi/2$ $\uparrow$



- Reactance (x_c) — $\boxed{x_c = \frac{1}{\omega C}}$

→ Inductor

$V - L\frac{di}{dt} = 0$



$\Rightarrow$ $V = L i_0 \omega \cos \omega t$

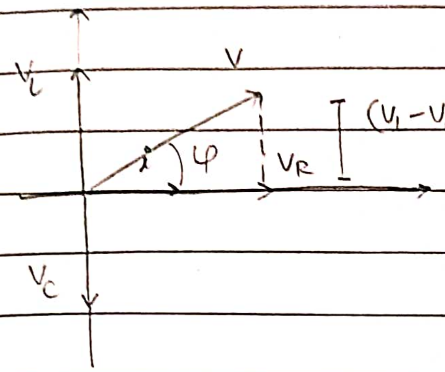
$\Rightarrow$ $V = i_0 (\omega L) \sin\left(\omega t + \frac{\pi}{2}\right)$

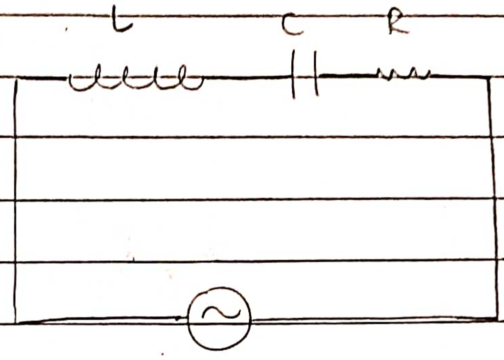
$\Rightarrow$ V leads i by $\pi/2$ $\uparrow$

- Reactance (x_L) — $\boxed{x_L = \omega L}$

REMARK: Reactance is a sort of resistance i.e. it does hinder current flow, yet circuit elements with reactance do not consume power.

L-C-R SERIES CIRCUIT





components in series
$\Downarrow$
current in phase

$$\boxed{V = \sqrt{V_R^2 + (V_L - V_C)^2}}$$

$$Z = \frac{V}{i} = \sqrt{\left(\frac{V_R}{i}\right)^2 + \left(\frac{V_L}{i} - \frac{V_C}{i}\right)^2} \Rightarrow \boxed{Z = \sqrt{R^2 + (x_L - x_C)^2}}$$

$$\tan(\varphi) = \frac{V_L - V_C}{R} = \left(\frac{x_L - x_C}{R}\right) \rightarrow c\varphi = \frac{R}{\sqrt{R^2 + (x_L - x_C)^2}} = \frac{R}{Z}$$

$$P = V_{rms} \cdot i_{rms}\, c\varphi = V_{rms} \cdot i_{rms} \cdot \frac{R}{Z}$$
$$= i_{rms}^2 R$$

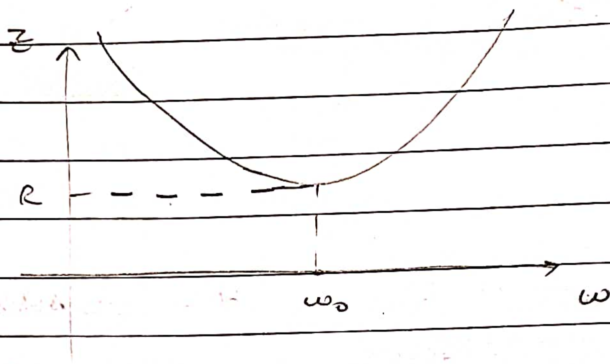
→ Series Resonance

$$Z = \sqrt{R^2 + (x_L - x_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}.$$

$$Z_{min} = R \quad \text{at} \quad \omega = \frac{1}{\sqrt{LC}}$$

This ω is called resonant/natural freq

$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$$



$\forall \; \omega < \omega_0$, $x_C > x_L \Rightarrow$ i leads V

$\forall \; \omega > \omega_0$, $x_L > x_C \Rightarrow$ V leads i

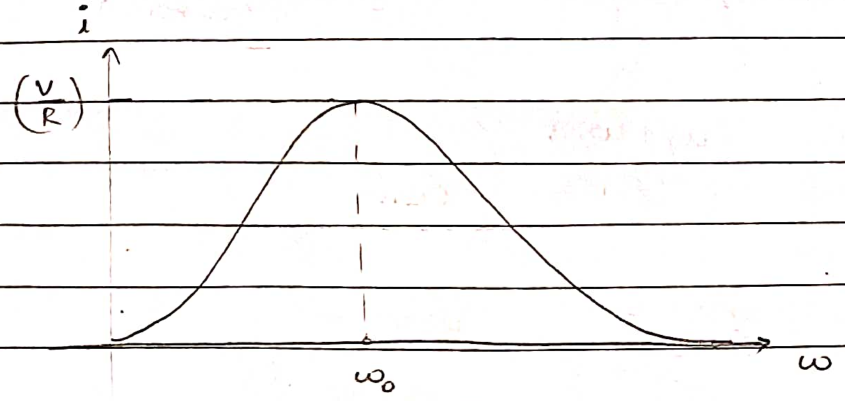
if $V = V_0 \sin\omega t$ & $\tan\varphi = \left(\frac{x_L - x_C}{R}\right)$

$\Rightarrow i = \left(\frac{V_0}{Z}\right)\sin(\omega t - \varphi)$

($\because x_L > x_C \Rightarrow$ V leads i by φ

In resonance,

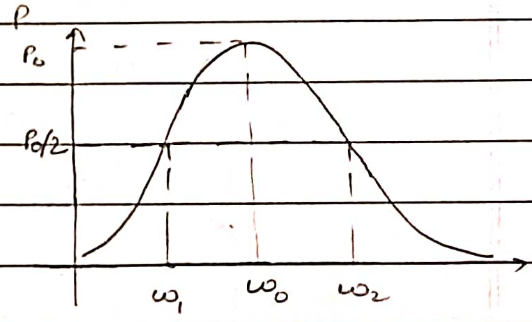
- i & V in same phase i.e $\tan\varphi = 0$
- $Z = Z_{min}$
- $P = P_{max}$
- $i = i_{max}$



- **Sharpness of resonance** – How easily resonance freq is distinguishable from neighbouring freq.

- **Quality factor (Q-factor)** –

$$\boxed{Q = \frac{\omega_0}{\underbrace{\Delta\omega}_{\text{(Bandwidth)}}}}$$



Bandwidth of resonance $\Delta\omega = (\omega_2 - \omega_1)$

at $\omega = \omega_1$ & ω_2

$P = P_0/2$

& $i = i_0/\sqrt{2}$

$$\sqrt{2}R = \sqrt{R^2 + \left(\frac{1}{\omega_1 C} - \omega_1 L\right)^2}$$

$$\Rightarrow \quad \frac{1}{\omega_1 C} - \omega_1 L = R \quad \text{—(i)} \qquad (\omega_1 < \omega_0 \Rightarrow x_C > x_L)$$

Similarly, $\omega_2 L - \frac{1}{\omega_2 C} = R$ —(ii) $(\omega_2 > \omega_0 \Rightarrow x_C < x_L)$

$(i)-(ii) \Rightarrow \frac{1}{C}\left(\frac{1}{\omega_1}+\frac{1}{\omega_2}\right) = L(\omega_1+\omega_2)$

$\Rightarrow (\omega_1+\omega_2)\left(\frac{1}{C\omega_1\omega_2} - L\right) = 0$

$\omega_1+\omega_2 \neq 0 \Rightarrow \omega_1\omega_2 = \frac{1}{LC}$

$(i)+(ii) \Rightarrow \frac{1}{C}\left(\frac{1}{\omega_1}-\frac{1}{\omega_2}\right) + L(\omega_2-\omega_1) = 2R$

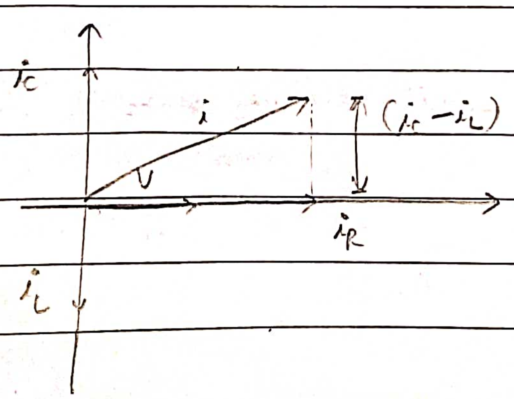
$\Rightarrow \omega_2-\omega_1 = \left(\frac{R}{L}\right)$

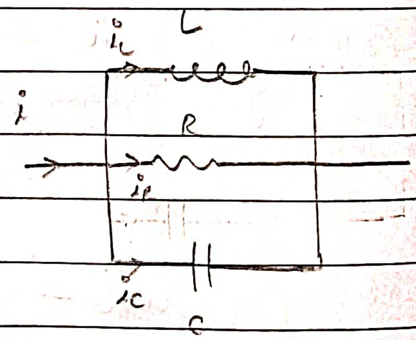
$\Rightarrow \boxed{Q = \frac{\omega_0 L}{R}}$

$= \frac{x_{L_0}}{R} = \frac{x_{C_0}}{R}$ (Reactance at resonance)

PARALLEL L-C-R CIRCUIT

Here, voltage in same phase

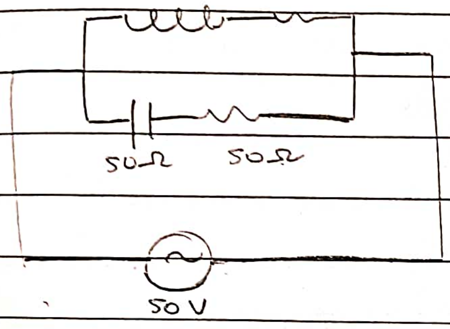




$$\boxed{i = \sqrt{i_R^2 + (i_C - i_L)^2}}$$

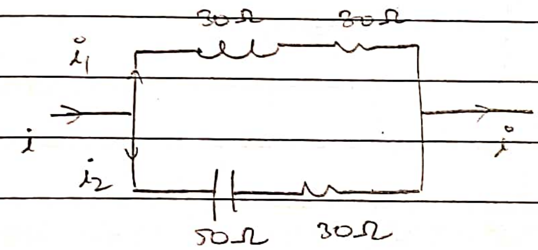
$$\Rightarrow \frac{V}{Z} = \sqrt{\left(\frac{V}{R}\right)^2 + \left(\frac{V}{x_C} - \frac{V}{x_L}\right)^2} \quad \Rightarrow \quad \boxed{\frac{1}{Z} = \sqrt{\frac{1}{R^2} + \left(\frac{1}{x_C} - \frac{1}{x_L}\right)^2}}$$

Q.



Find current in circuit & impedence of circuit

A.

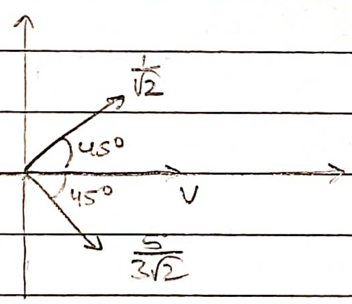


$$Z_{LR} = \sqrt{R^2 + x_L^2} = 30\sqrt{2}$$

$$Z_{RC} = \sqrt{R^2 + x_C^2} = 50\sqrt{2}$$

$$i_{LR} = \frac{V}{Z_{LR}} = \frac{5}{3\sqrt{2}}$$

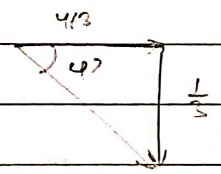
$$i_{RC} = \frac{V}{Z_{RC}} = \frac{1}{\sqrt{2}}$$



$$\tan\phi_{LR} = \frac{x_L}{R} = \frac{30}{30} = 1$$

$$\Rightarrow \phi_{LR} = \frac{\pi}{4} \Rightarrow i_1 \text{ lags } V \text{ by } \pi/4$$

$\equiv$



$$\tan\phi_{RC} = \frac{x_C}{R} = \frac{50}{50} = 1$$

$$\Rightarrow \phi_{RC} = \frac{\pi}{4} \Rightarrow i_2 \text{ leads } V \text{ by } \pi/4$$

$$i = \frac{\sqrt{17}}{3}$$

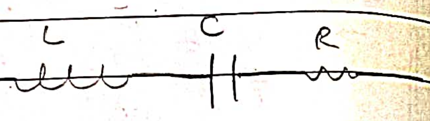
$$Z = \frac{V}{i} = \frac{150}{\sqrt{17}}$$

→ Complex impedance Method

$R \rightarrow R$

$x_L \rightarrow jx_L$

$x_C \rightarrow -jx_C$



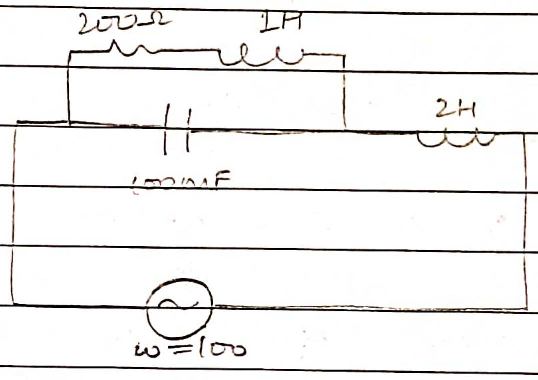
where $j = \sqrt{-1}$

Now, treat all components as resistors.

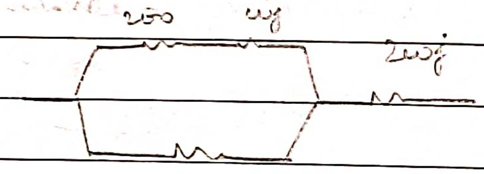
So, $Z = R + j\omega L - \dfrac{j}{\omega C}$

(complex impedence)

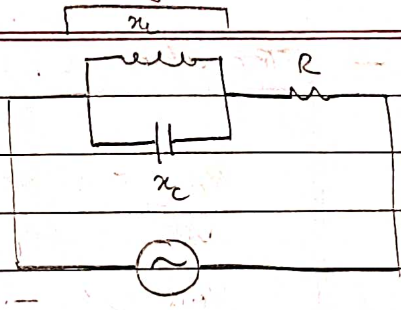
$$\boxed{\begin{pmatrix}\text{Impedence}\\ \text{of Circuit}\end{pmatrix} = |Z|}$$

Q. 

Find impedance.

A. $\equiv$ 

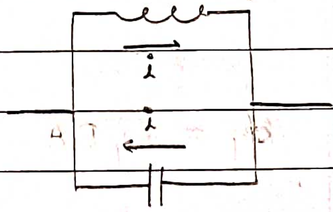
$$Z = \left| \frac{1}{\left(\frac{1}{200+100j}\right) + (10j)} + 200 \right|$$



NOTE: $\dfrac{1}{Z} = \dfrac{1}{x_C} - \dfrac{1}{x_L}$

In resonance, $Z = \infty$

So, no current flows through circuit.



Hence, this combination behaves as an open switch.

$\Rightarrow$ Net current flown $= 0$

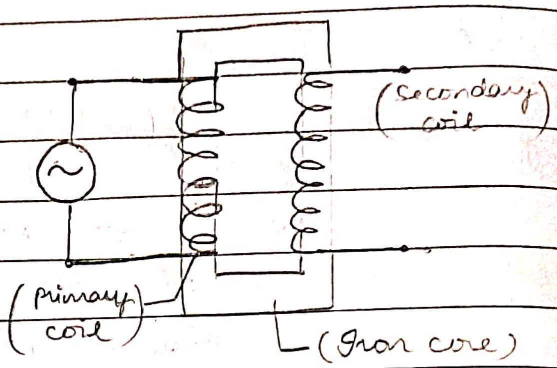
AC INSTRUMENTS

→ Hot wire (Voltmeter & Ammeter)

- Based on heating effect of current
- Graduations of scale are not uniform since i^2 or V^2 is being measure instead of i or V

→ Transformer

Semantic Diagram



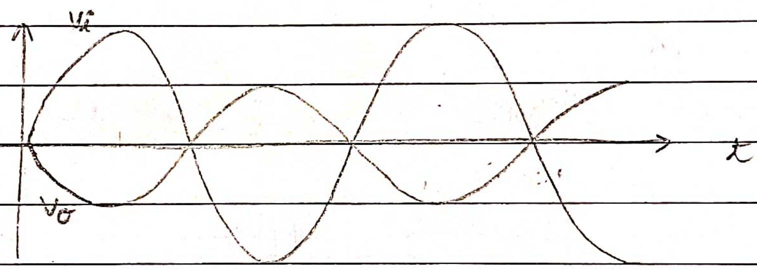
$\frac{V_{out}}{V_{in}} < 1 \Rightarrow$ step-down

$\frac{V_{out}}{V_{in}} > 1 \Rightarrow$ step-up

$$\varphi_P = N_P BA \Rightarrow V_i = \frac{d\varphi_P}{dt} = N_P A \frac{dB}{dt}$$

$$\varphi_S = N_S BA \Rightarrow V_o = -\frac{d\varphi_s}{dt} = -N_S A \frac{dB}{dt}$$

$$\boxed{V_o = -\left(\frac{N_S}{N_P}\right) V_i}$$



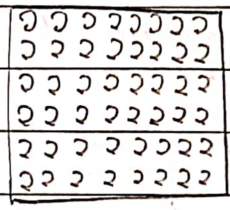
Ideally, $P_{in} = P_{out}$

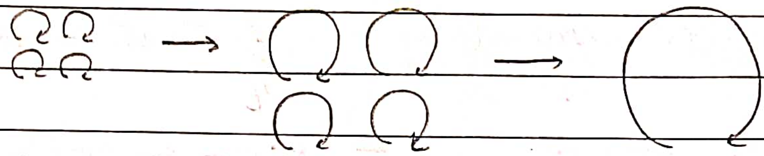
But a fraction of power is lost due to :-

1) Joule heating
2) Magnetic hysteresis loss
3) Eddy currents

- Eddy currents – Small loops of current form in the core as flux through it changes

Eddy currents in boundaries of neighbouring loops combine to form larger loops of current

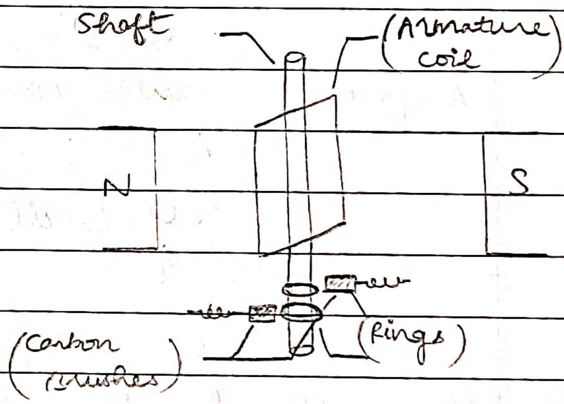




→ AC & DC Generator

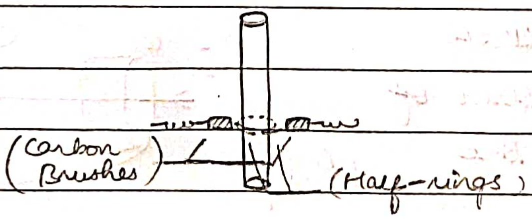
$$\varphi = NBA\cos\omega t$$

$$V = -\frac{d\varphi}{dt} = NBA\sin\omega t$$



AC Generator

To convert this to DC Generator, we replace 2 full rings with 2 half-rings. This set-up gives uni-directional current



EM WAVES

→ Maxwell's Eqⁿs

1) Gauss Law : $\oint_S \vec{E} \cdot d\vec{S} = \frac{q_{enclosed}}{\epsilon_0}$

2) $\oint_S \vec{B} \cdot d\vec{S} = 0$

3) Ampère's Circuital Law : $\oint_L \vec{B} \cdot d\vec{l} = \mu_0 \, i_{threaded}$

4) Faraday's Law : $\oint \vec{E} \cdot d\vec{l} = -\left(\frac{d\phi}{dt}\right)$

→ Inconsistency in Ampere's Law

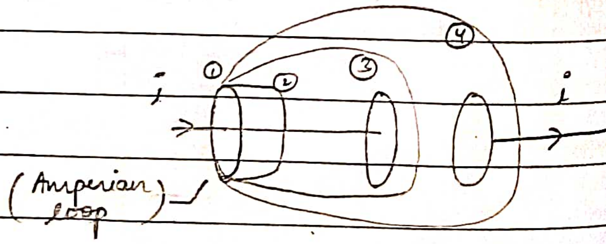
Ampere's law can be written as

$$\oint_L \vec{B} \cdot d\vec{l} = \mu_0 \int_S \vec{J} \cdot d\vec{A}$$

where the surface S over which the flux of J is evaluated can be any open surface bounded by the Amperian loop L.

Consider a || plate cap. being charged

For each of these surfaces, the flux of J should give the same current



As evident, surfaces 1, 2, 4 are pierced by the current i.
However, no current passes through 3.

To remove this inconsistency, we propose the existence of a displacement current 'flowing' b/w the plates of the cap.

This disp. current should be equal to the conduction current (i) entering the surface 3.

$$\oint_{S_1} \vec{J} \cdot d\vec{A} = i = \frac{dq}{dt} = \frac{d}{dt}\left(\epsilon_0 \oint_S \vec{E} \cdot d\vec{A} \right)$$

$$= \epsilon_0 \frac{d\varphi_E}{dt}$$

$\therefore$ If we define $i_d = \epsilon_0 \frac{d\varphi_E}{dt}$

& $i_{Total} = i + i_d$

Ampere's law becomes

$$\boxed{\oint_L \vec{B} \cdot d\vec{l} = \mu_0 i_0 + \mu_0 \epsilon_0 \frac{d\varphi_E}{dt}}$$

As a consequence of this modification, changing $\vec{E}$ can produce $\vec{B}$

Changing $\vec{B}$ produces $\vec{E}$
Changing $\vec{E}$ produces $\vec{B}$

If $\vec{E}$ / $\vec{B}$ is produced in space, the disturbance is carried forward.

This is known as EM wave.

- **Wavefront** — A surface where all waves are in same phase at a particular time.

Dirxⁿ of propogation of wave is normal to wavefront.

$$\left.\begin{aligned}\vec{E} &= E_0 \sin(\omega t - kx)\,\hat{e}_1 \\ \vec{B} &= B_0 \sin(\omega t - kx)\,\hat{e}_2\end{aligned}\right\}$$ In phase, where $\hat{e}_1 \cdot \hat{e}_2 = 0$

$\vec{E}$, $\vec{B}$, $\vec{c}$ are mutually $\perp$

$\vec{c}$ ⟶ (dirxⁿ of propogation of EM wave)

So, $\hat{c} = \hat{E} \times \hat{B}$

E_0 & B_0 are related by $\boxed{\dfrac{E_0}{B_0} = c}$

where $\boxed{c = \dfrac{1}{\sqrt{\mu_0 \epsilon_0}}}$ in vacuum

For a medium, $\boxed{c = \dfrac{1}{\sqrt{\mu_0 \mu_r \epsilon_0 \epsilon_r}}}$

ϵ_r – relative Permittivity

μ_r – relative Permeability

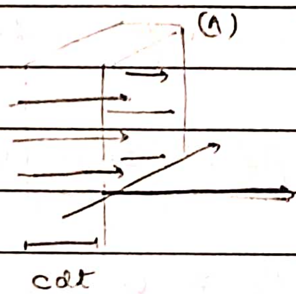
$\Rightarrow \boxed{c' = \dfrac{c}{\sqrt{\mu_r \epsilon_r}} = \dfrac{c}{n}}$

n – (Refractive index)

• Avg. Energy density – $\dfrac{\int \left(\dfrac{\epsilon_0 E^2}{2} + \dfrac{B^2}{2\mu_0}\right) dV}{\int dV}$

$\Rightarrow \left(\dfrac{1}{2}\right)\left(\dfrac{\epsilon_0 E_0^2}{2} + \dfrac{B_0^2}{2\mu_0}\right) = \dfrac{1}{2}\epsilon_0 E_0^2$

$= \dfrac{B_0^2}{2\mu_0}$



Power transferred

$= (A\,c\,dt)\left(\dfrac{1}{2}\epsilon_0 E_0^2\right)$

• Intensity – Power transferred per unit normal area

$\Rightarrow \boxed{I = \dfrac{1}{2} c\,\epsilon_0 E_0^2}$

$= \dfrac{1}{2}\dfrac{E_0 B_0}{\mu_0}$

- **Poynting vector** –

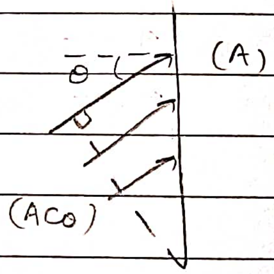
$$\boxed{\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0}}$$
$$= \vec{E}\times\vec{H}$$

Poynting vector contains info. about instantaneous power transmitted & dirxn of propogation of wave.

- **Force exerted by EM wave** –

(Momentum of wave) $\boxed{p = \frac{E}{c}}$ ← Energy, ← speed of light

Let us consider an area absorbing light



$$p_i = \left(\frac{IAC\theta}{c}\right)dt$$

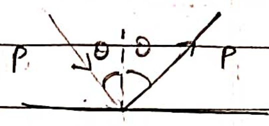
$$p_f = 0$$

$$\Rightarrow \frac{dp}{dt} = \left(\frac{IA}{c}\right)C\theta \Rightarrow \boxed{F_{abs} = \left(\frac{IA}{c}\right)C\theta}$$

{This force is directed along dirxn of wave.}

If the area was reflecting light instead,

$$dp = 2p\,c_\theta\,dt = 2\left(\frac{IA\,c_\theta}{c}\right) c_\theta\,dt$$



$$\Rightarrow \frac{dp}{dt} = \frac{2IAc_\theta^2}{c}$$

$$\Rightarrow \boxed{F_{ref} = \frac{2IAc_\theta^2}{c}}$$

{ This force is directed along <u>normal</u> to <u>surface</u> }

- Radiation/Radiant Pressure –

$$P_{abs} = \frac{F_{abs}\,c_\theta}{A} = \left(\frac{I\,c_\theta^2}{c}\right) \qquad \left[\because\ P = \frac{F_\perp}{A}\right]$$

$$P_{ref} = \frac{F_{ref}}{A} = \left(\frac{2Ic_\theta^2}{c}\right)$$